1.2 Graph Drawing Techniques

Graph drawing is the automated layout of graphs
We shall overview a number of graph drawing techniques

For general graphs:

Force Directed

- Spring Embedder
- Barycentre based

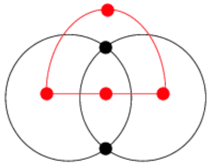
Multicriteria optimization

For specific graph types:

Planar

Hierarchical

Orthogonal



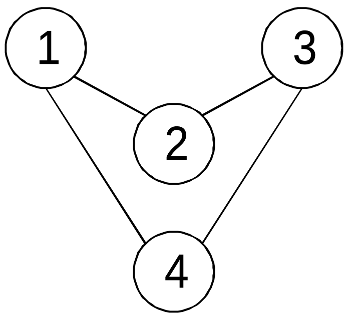
Graph Drawing is Application Specific

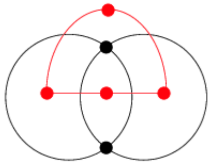
Task

Keep the graph theoretic structure of the graph and map the following applications to the graph:

1. As a UML diagram with 1: 'Person', 2: 'Teacher', 3: 'Class', 4: 'Student'. Person is a generalization of both Teacher and Student. The edges connecting Class to Teacher and Student are associations.
2. As a representation of a round trip travel plan. 1: ‘Corvallis', 2: ‘Miami', 3: ‘New York', 4: ‘Los Angeles'

Redraw the graph in a good way for each application





Force Directed Graph Drawing Methods

A design for layout of graph data structures
Eades' Spring Embedder

P. Eades: 'A Heuristic for Graph Drawing'. Congressus Numerantiom 42, 1984. pp. 149-60.

Here we will look at force directed approaches in general and overview of energy systems

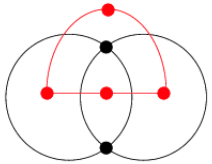
Principles of Spring Embedding

A heuristic approach

We need to calculate

1. An attractive force on each vertex, treating edges as springs, forcing the vertices together
2. A repulsive force on each vertex, treating vertices as charged particles and calculated from distance to each other vertex

Then we move each vertex with the balance of the total force

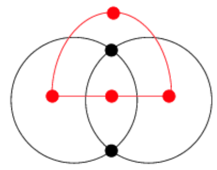


Algorithm

```
SPRING(G:graph)
place vertices of G in random locations
repeat M times
      calculate the force on each vertex
      move the vertices f*(force each vertex)
draw graph
```

If d is distance between two vertices
Attractive force = $-k*d$
Repulsive force = r/d^2

Constants k, r, f, M need to be set by the implementer
(typically through trial and error)



Notes on the algorithm

Each force calculation is the sum of each edge connection and every other vertex. Consider it a vector - it needs direction + amount

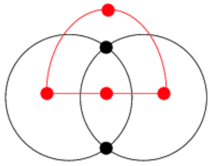
Time complexity $O(M*N^2)$ because calculating distance between all vertices is N^2

Hookes law (linear force) used for edges, but Eades originally used a logarithm

Inverse square force for vertices - less repulsion when further apart

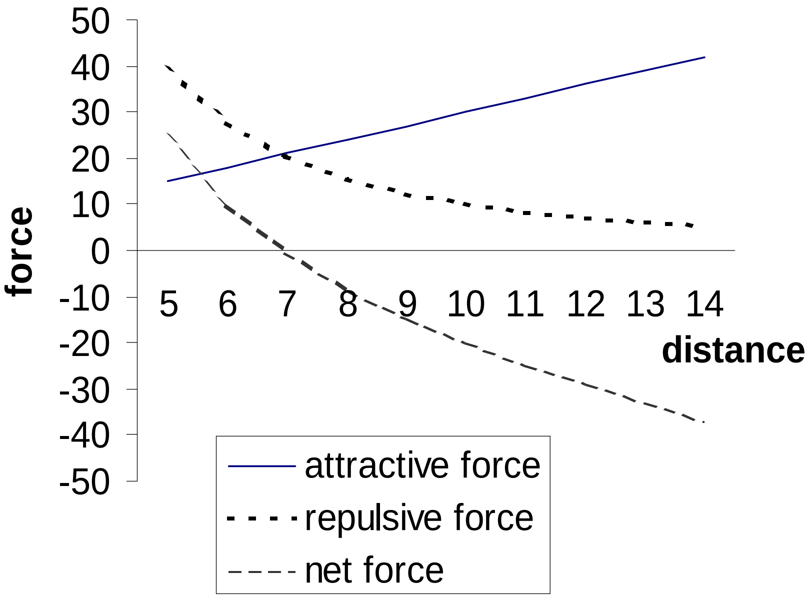
Question

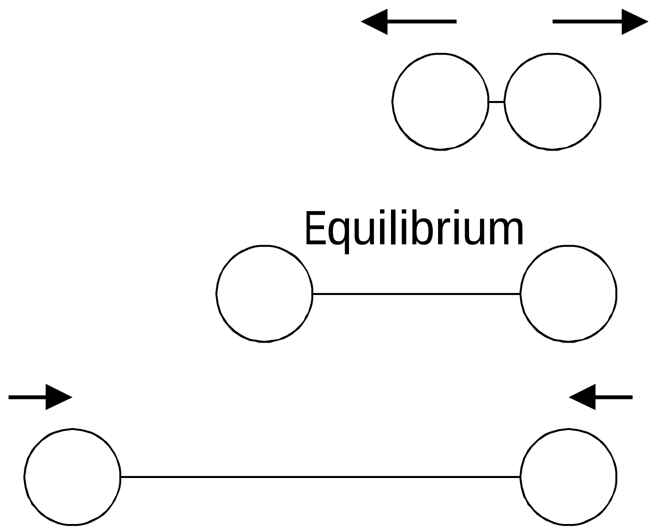
Why have most implementations moved from a logarithmic edge force to a linear one?

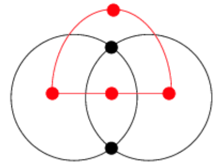


How the forces work

Length and direction of arrows indicate the force on a vertex. Here for only 2 vertices



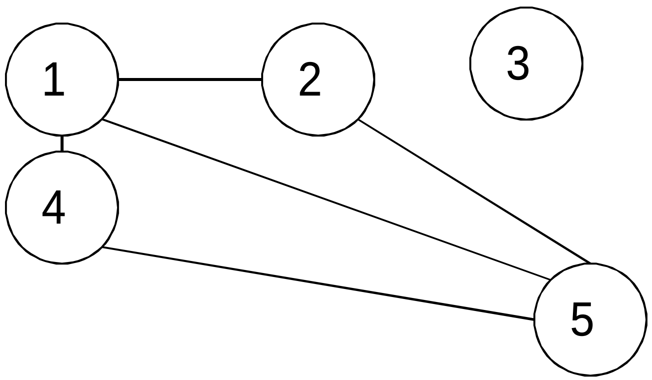




Exercise

Task

Estimate the movement of vertices in this graph for one iteration of a spring embedder
Guess a final layout after 100 iterations

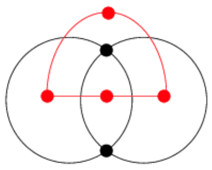


Generalizing force directed approaches

A force directed graph drawing system consists of:

1. Model: a force system calculating force based on the vertices and edges
2. Algorithm: a method for finding the equilibrium state of the force system, i.e. where the total force on each vertex is 0

The trick is to find a force system that is both quick to calculate and forms a good graph layout



Other force directed methods

Barycenter based approach

force moves vertices towards their barycenter, the average position of neighbours

Force based on a graph theoretic notions

attempt to get distance between vertices proportional to graph theoretic distance

Magnetic field ideas

parallel, radial and concentric fields

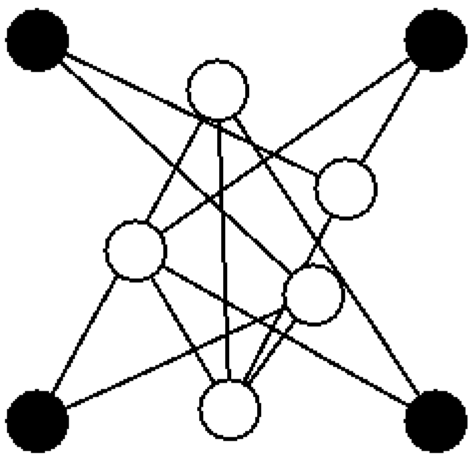
Energy functions that use aesthetic criteria

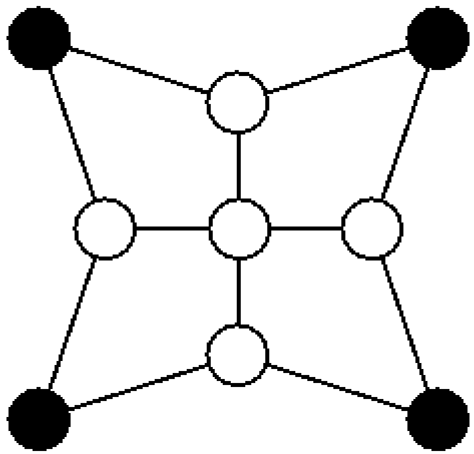
Barycentre example

Several iterations of moving vertices to the average position of their neighbours

Needs some fixed vertices or all the vertices will end up at the same position

Works well for quick drawing of very symmetric graphs (and that have got obvious fixed vertices)





Pros and cons of force directed methods

Pros

Works well on sparse, smallish graphs

Easy to understand due to force analogy

Quick to implement

Can be used effectively with other methods

Cons

Different results with the same graph

Use with straight line, connected graphs only

Indirect definition of good aesthetics

Multicriteria optimization for graph drawing

Graph Drawing is just another optimization problem

Hence if we can measure the quality of a layout, we should be able to apply standard search mechanisms to improve the quality of the layout. Current search methods used:

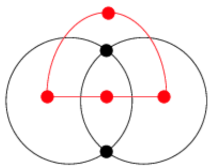
- Simulated Annealing
- Hill Climbing
- Genetic Algorithms

Aesthetic metrics for graphs

edge crossing (total)
edge length (total, or variance)
edge bends (total)
graph size/aspect ratio (various ratios possible)
vertex separation (e.g. variance of distance to nearest neighbour)
angular resolution

- This attempts to avoid very small angles
- Measured by variance of inverse angle
- Threshold often used

Application specific measures can be defined.



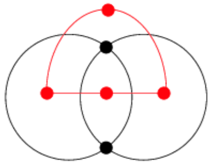
Putting the multicriteria optimizer together

Measures are combined in a weighted sum
The weights are used to normalize the measures, and define priorities

In a simulated annealing approach, successive alterations are made to the graph (for instance, random vertex movement) and improvements to the fitness mean the change is kept.

- In an effort to avoid local minima, some bad moves are also retained.

Criteria are not orthogonal and so care needs to be taken when assigning weights



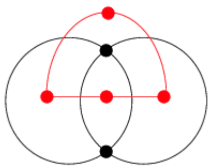
Planar graph layout

Some graphs can be guaranteed to be laid out without edge crossings

There are fast algorithms (linear time) to detect planar graphs and generate plane layouts

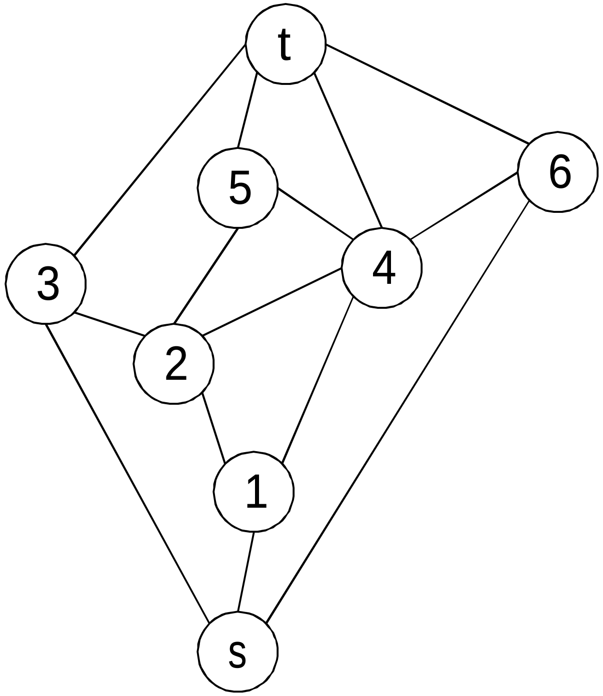
- Interestingly, the problem of finding the minimum number of crossings for a non-planar graph is NP-Complete

However, plane layouts often need improvement, through e.g. Force directed methods



Example plane layout

Often a planar layout algorithm relies on numbering vertices between a source and target.

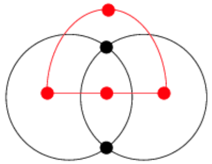


Hierarchical drawing

Polynomial time complexity for graphs without cycles.
Typically relies on layout on an integer grid
First, assign layers to vertices
- Simple layer assignment (shortest depth) relies on placing vertices in their first available layer, which can lead to ‘layer bloat’

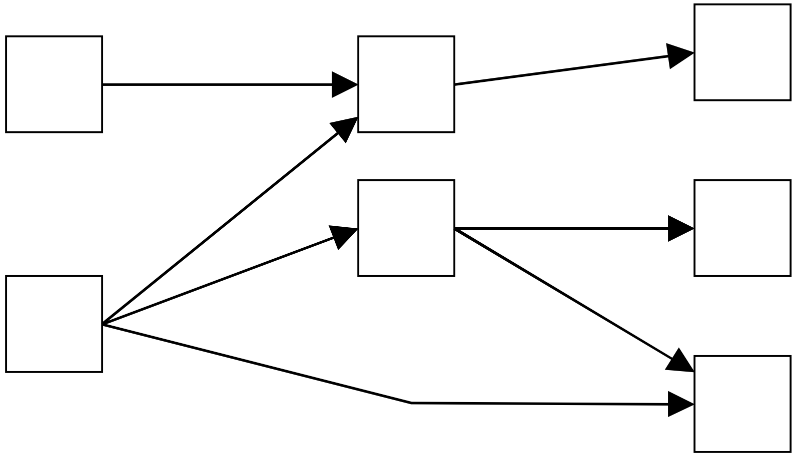
Second, assign vertices to locations on layers. Various mechanisms are possible:
- The x-barycentre of parents or children vertices
- Minimal crossing assignment
- The method must address conflict resolution

Dummy vertices are used when an edge crosses a level



Hierarchical example

Here we use a left to right approach to layout

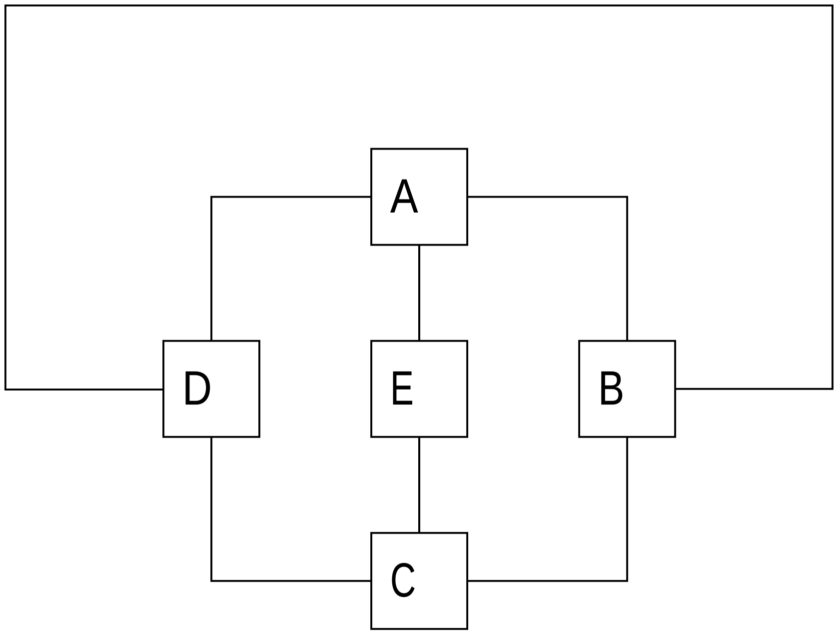


Orthogonal layout

Here, edges run only vertically or horizontally in a plane graph

Typical layout of a PCB, and some software engineering diagrams

Various algorithms generate layout in fast time

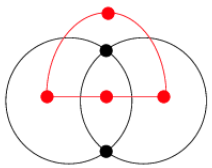


Using specialist techniques for general graphs

If we have a ‘nearly’ planar graph, or a diagram with a sense of flow, but some cycles, we can:

Planarize a graph by adding new vertices or removing edges, then once laid out return the diagram to the previous topology

Make a graph cycle free by reversing edge direction, again returning the layout to the previous topology



Summary

The Spring Embedder produces a fairly symmetric graph with even, short edge length and an even vertex distribution that is fairly easy to implement

Multicriteria approaches are more flexible, but typically take longer to run and is more effort to implement

Hierarchical, Planar and Orthogonal layout methods run quickly, but only on specific graph types